An *interval estimator* is a rule specifying the method for using the sample measurements to calculate two numbers that from the endpoints of the interval. One or both of the endpoints of the interval, being functionals of the sample measurements, will vary randomly from sample to sample. Because of this reason, the length and the location of the interval are random quantities, and we cannot be sure that the target parameter θ (which is fixed) will fall between the endpoints of any single interval calculated from a single sample. Our goal is to find an interval estimator capable or generating narrow intervals that have a high probability of containing θ.

Interval estimators are also called *confidence intervals.* The upper and lower endpoints of a confidence interval are called *upper* and *lower confidence limits*, respectively. The probability that a (random) confidence interval will enclose θ is called the *confidence coefficient.*

Suppose that $\hat{\theta}_L$ and $\hat{\theta}_U$ are the (random) lower and upper confidence limits, respectively, for a parameter θ. Then, if

$$P(\hat{\theta}_L \leq \theta \leq \hat{\theta}_U) = 1 - \alpha,$$

the probability $(1 - \alpha)$ is the *confidence coefficient.* The resulting random interval given by $[\hat{\theta}_L, \hat{\theta}_U]$ is called a *two-sided confidence interval.*

it is also possible to form *one-sided confidence interval* such that

$$P(\hat{\theta}_L \leq \theta) = 1 - \alpha.$$

In this case, the confidence interval is $[\hat{\theta}_L, \infty)$. Similarly, we could have an upper one-sided confidence interval such that

$$P(\theta \leq \hat{\theta}_U) = 1 - \alpha.$$

In this case, the confidence interval is $(-\infty, \hat{\theta}_L]$.

In the previous note, we discussed some unbiased point estimators for the parameters μ, p, $\mu_1 - \mu_2$, and $p_1 - p_2$. For large samples, all these point estimators have approximately normal sampling distributions. Please take a look at the table given in the previous note for their standard errors.

if the target parameter θ is μ, p, $\mu_1 - \mu_2$, or $p_1 - p_2$, then for large samples,

$$Z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}}$$

possesses approximately a standard normal distribution.

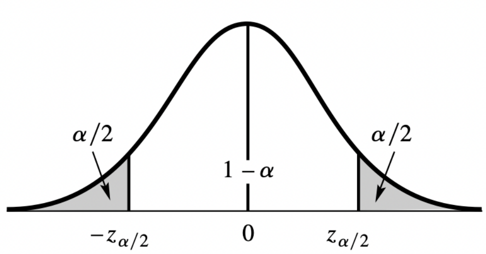
Let $\hat{\theta}$ be a statistic that is normally distributed with mean θ and standard error $\sigma_{\hat{\theta}}$. Find a confidence interval for θ that possesses a confidence coefficient equal to $(1 - \alpha)$.

The quantity

$$Z = \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}}$$

has a standard normal distribution. Let's select two values $-z_{\alpha/2}$ and $z_{\alpha/2}$, such that

$$P(-z_{\alpha/2} \leq Z \leq z_{\alpha/2}) = 1 - \alpha.$$



Then we have

$$P(-z_{\alpha/2} \leq \frac{\hat{\theta} - \theta}{\sigma_{\hat{\theta}}} \leq z_{\alpha/2}) = 1 - \alpha,$$

which implies

$$P(\hat{\theta} - z_{\alpha/2}\,\sigma_{\hat{\theta}} \leq \theta \leq \hat{\theta} + z_{\alpha/2}\,\sigma_{\hat{\theta}}) = 1 - \alpha.$$

Thus the endpoints for a $100(1-\alpha)\%$ confidence interval for θ are given by

$$\hat{\theta}_L = \hat{\theta} - z_{\alpha/2}\,\sigma_{\hat{\theta}} \;\text{ and }\; \hat{\theta}_U = \hat{\theta} + z_{\alpha/2}\,\sigma_{\hat{\theta}}.$$